**Mathematics Department**

**Year 12 Mathematical Methods 2022**

**Problem Solving and Modelling Task**

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| **Student Name**: |  | **Student Number:** |  |
| **Teacher**: |  | | |

**Assessment objectives**

This assessment instrument is used to determine student achievement in the following objectives:

1. Select, recall and use facts, rules, definitions and procedures drawn from Unit 3 Topics 2 and/or 3.
2. Comprehend mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3.
3. Communicate using mathematical, statistical and everyday language and conventions.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions by explaining mathematical reasoning.
6. Solve problems by applying mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3.



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| **Subject**: | Mathematical Methods | | | **Date of Issue:** | | | 1 February 2022 | | |
| **Technique:** | Problem Solving and Modelling Task | | | **Due Date:** | | | 1 March 2022 | | |
| **Unit:** | 3 Further Calculus | | | **Weighting:** | | | 20% | | |
| **Topic:** | * Calculus/Functions | | | | | | | | |
| **Conditions** | | | | | | | | |
| **Response Type** | | Written report | | | **Length** | | | 10 pages (Max)  Maximum 2000 words excluding appendixes |
| **Duration** | | 4 Weeks (Including 3 hours of class time) | | | **Individual/Group** | | | Individual |
| **Resources Available** | | The use of Technology is required, eg:   * Graphics Calculator * Graphing/spreadsheet program **(USE ONLY DESMOS)** * Access to the Internet | | | | | | |
| **Criterion** | | | **Marks Allocated** | | | **Result** | | |
| **Formulate** | | | **4** | | |  | | |
| **Solve** | | | **7** | | |  | | |
| **Evaluate and Verify** | | | **5** | | |  | | |
| **Communicate** | | | **4** | | |  | | |
| **Total:** | | | **20** | | |  | | |

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| **Context** |
| Tortoises, start your engines for the Inaugural Zootopia 12000, proudly sponsored by AWC Corporate Partners Terra Australis and Indigie Arts.  The Inaugural Zootopia 12000 is an extraordinary tortoise racing event to be introduced at Adelaide Zoo’s annual Zootopia Celebrations on October 25th. The concept was designed to highlight the plight of the Western Swamp Tortoise; Australia’s smallest freshwater tortoise. This now endangered reptile was once commonplace in the swamps and rivers surrounding the city of Perth, Western Australia. However, as a result of the introduction of non-native feral predators the number of Western Swamp Tortoises in the wild plummeted to dangerously low numbers <https://www.zoossa.com.au/wp-content/uploads/Freshwater-Tortoises-Oct-09.pdf>. In a joint mission, involving both the Perth and Adelaide Zoos together with Australian Wildlife Conservation (AWC) partners, a plan was conceived to protect this species from extinction and eventually revitalise numbers in the wild. Capitalising on the Australian love of competitive sports it was decided to include a robotic tortoise race. The race, in which live animals are not harmed or exploited, is designed to excite and inspire wildlife supporters to help fund research and breeding efforts of the Western Swamp Tortoise.  Experienced racing commentator Bruce McAvaney and Grand Prix Champion Mick Doohan will begin their pre-race commentary leading up to the parade lap at 11:30am. Following the parade, Mick Doohan will also serve as Grand Marshal to start the race with the green flag and later wave the checkered flag to signal the winner!  It’s always exciting to cheer on the robot tortoises as they race down the straight track! Truly with cleverly designed robotic tortoises, anything can happen. Any two of the 18 species of mechanical tortoises could qualify for the final, ultimate Zootopia 12000. Each robotic tortoise is faithfully designed and produced to mimic the real-life athletic capabilities and idiosyncratic behavioural characteristics of the various species they represent. The contestants are as real to life as current technology allows, no expense has been spared in their development.  The tortoise race is expected to be one of the most popular events and has been highlighted by Top Events Australia website as one of the unmissable events in South Australia! Come out and see why the “Greatest Spectacle in Tortoise Racing” will become such a beloved annual zoo event. |

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| **Task** |
| Your task is to design possible models to be uploaded to the tortoises for the two final competitors in the Zootopia 12000. The models for each tortoise must reflect different constraints, obviously as a result of the tortoises’ physical limitations and individual idiosyncrasies. The race will, however, be designed to be close. There must be one hundredth of a second between them. To maximise gambling revenue the race should be designed to be thrilling from start to finish. Patrons must be kept on the edge of their seat, where the race could be any tortoises’, thereby generating a betting frenzy from the punters.  You will submit a report that outlines the development of the models; including a graph of the race and all relevant computations – all functions must be fully developed with full mathematical justification and reasoning.   * The race must be thrilling and consequently should be designed to elicit a high degree of uncertainty as to which competitor will be the ultimate victor. * The length of the race is between 20 to 30 metres. * The velocity of any tortoise cannot exceed 6metres/minute. * Each tortoise should be identified by a unique name. * Each tortoises’ race will be represented by a piecewise function where all functions must be fully developed with mathematical justification and reasoning. * The race must incorporate a minimum of four different families of functions of which two may be polynomial, the remaining functions must be selected from logarithmic, exponential, periodic and hyperbolic functions. * The first tortoise must include a section of the race where its velocity is constant. * The second tortoise must include a section of the race where its acceleration is constant. * Use appropriate mathematical methods to demonstrate that the graphs of the tortoise finalists are smooth throughout the race.   \*Whole race should be constructed on desmos. |

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| **Checkpoints:** |  |
| **Weekly checkpoints during class time:**  Student’s progressed will be checked during 3 hour lessons. The checkpoints are: | |
| Week 3 (formulate) | Date: 8th February 2022 |
| Week 6 (solve + verify and evaluate) – Draft feedback | Date: 15th February 2022 |
| Week 7 Preliminary submission | Date: 22nd February 2022 |

**An approach to Problem Solving and Mathematical Modelling**

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|  | Design a plan to translate the problem into a mathematically purposeful representation by first:   * Completing any necessary research to formulate a plan. * Determining applicable mathematical concepts, techniques and technology that are required to model the problem. * Deciding on the design of the tortiose race you want to model. * Identifying and documenting appropriate assumptions, variables and observations, prior to modelling the race based on the logic of proposed models: must include a description of how the parameters for the race functions and the model will be determined. * Considering how you will incorporate the use of technology to assist with your strategy. |
| Select and apply mathematical and graphical procedures, concepts and techniques previously learned to solve the mathematical problem to be addressed through your model (you may reconsider and/or modify initial variables, observations and assumptions to ensure that the problem can be solved).   * Analysis of the shape of each section of the race and the selection of appropriate functions to model them. * Models should satisfy the constraints and address task specific criteria. * Solutions should be found using algebraic, graphical, arithmetic and numeric methods, with and without technology. |
| Once a possible solution has been achieved, consider the reasonableness of the solution and/or the utility of the model in terms of the problem posed. Are there any other variables to be considered, does your choice of model need to be revised or your functions refined?   * Evaluate your results and make a judgment about the validity of the solution. * Justify procedures used and decisions made by explaining your reasoning and providing full mathematical justification of the development of your model. * Verify your solution. * Assess the strengths and limitations of your final solution and include any constraints encountered or refinements made to functions and/or the final model. * This stage emphasises methodological rigour. |
| Communicate your findings systematically and concisely, using mathematical and everyday language.   * Draw conclusions, discussing the key results and the strengths and limitations of the solution. * Ensure your report has an introduction, body, conclusion and references, which can read independently of the task and that you solved the original problem posed. * You could offer further explanation, justification and recommendations, framed in the context of the initial problem. |

**Marking Guide**

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| **Criterion: Formulate** |  |
| **Assessment Objectives:**  1. **Select**, **recall** and **use** facts, rules, definitions and procedures  2. **Comprehend** mathematical concepts and techniques  5. **Justify** procedures and decisions by explaining mathematical reasoning | |
| **The student work has the following characteristics:** | **Marks** |
| * Documentation of appropriate assumptions * Accurate documentation of relevant observations * Accurate translation of all aspects of the problem by identifying mathematical concepts and techniques | **3 - 4** |
| * Statement of some relevant observations * Translation of simple aspects of the problem by identifying mathematical concepts and techniques | **1 - 2** |
| * Does not satisfy any of the descriptors above | **0** |

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| **Criterion: Solve** |  |
| **Assessment Objectives:**  1. **Select**, **recall** and **use** facts, rules, definitions and procedures  6. **Solve** problems by applying mathematical concepts and techniques | |
| **The student work has the following characteristics:** | **Marks** |
| * Use of complex procedures to reach an accurate solution * Discerning application of mathematical concepts and techniques relevant to the task * Efficient use of technology | **6 - 7** |
| * Use of complex procedures to reach a reasonable solution * Application of mathematical concepts and techniques relevant to the task * Appropriate use of technology | **4 - 5** |
| * Use of simple procedures to make some progress towards a solution * Simplistic application of mathematical concepts and techniques relevant to the task * Superficial use of technology | **2 - 3** |
| * Isolated use of technology or procedures relevant to the task | **1** |
| * Does not satisfy any of the descriptors above | **0** |

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| **Criterion: Evaluate and Verify** |  |
| **Assessment Objectives:**  4. **Evaluate** the reasonableness of solutions  5. **Justify** procedures and decisions by explaining mathematical reasoning | |
| **The student work has the following characteristics:** | **Marks** |
| * Justification and explanation of procedures used, and decisions made using mathematical reasoning * Evaluation of the reasonableness of solutions by considering the results, assumptions and/or observations * Evaluation of relevant strengths and limitations of the solution and/or model | **4 - 5** |
| * Explanation of procedures used, and decisions made * Statement about the reasonableness of solutions by considering the context of the task * Statement about relevant strengths and limitations of the solution and/or model | **2 - 3** |
| * Statement about procedures, decisions and/or the reasonableness of solutions | **1** |
| * Does not satisfy any of the descriptors above | **0** |

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| **Criterion: Communicate** |  |
| **Assessment Objectives:**  3. **Communicate** using mathematical, statistical and everyday language and conventions | |
| **The student work has the following characteristics:** | **Marks** |
| * Correct use of appropriate mathematical, statistical and everyday language and conventions to develop the response * Coherent and concise organisation of the response, appropriate to the genre, including a suitable introduction, body and conclusion, which can be read independently of the task sheet | **3 - 4** |
| * Use of some appropriate mathematical, statistical and everyday language and conventions to develop the response * Adequate organisation of the response | **1 - 2** |
| * Does not satisfy any of the descriptors above | **0** |